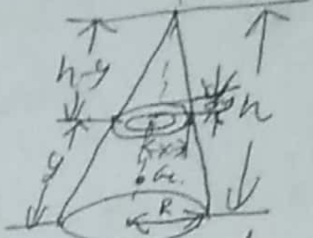
Unit-3 1) Centroid and Centroid of Gravity
2) Area Moment of Inertia

* Determine the centre of gravity of a right regular solid cone of radius 'R' and height h

A)



$$\frac{h}{h-y} = \frac{R}{x}$$

$$x = \frac{R(h-y)}{h}$$

Consider an elemental cone from whole cone.

$dm = \pi \rho \times V$ $\quad$ dy = width

$dM = \rho \times \pi x^2 \times dy$ $\quad$ x = radius of elemental cone

$= \rho \times \pi \frac{R^2 (h-y)^2}{h^2} dy$ $\quad$ It is at distance of y from base

y Centre of mass $= \frac{1}{M}\int_0^h y\,dm$ $\quad$ M = whole mass of cone, dm = mass of elemental cone

$= \frac{1}{M}\int_0^h y \frac{\rho \times \pi R^2 (h-y)^2}{h^2} dy$

$= \frac{\rho}{M} \frac{\pi R^2}{h^2} \int_0^h \frac{y(h-y)^2}{h^2} dy$ $\quad$ $\rho = \frac{M}{V}$

$= \frac{1}{V} \frac{\pi R^2}{h^2} \left[\int_0^h y(h^2 - 2hy + y^2)\,dy\right]$ $\quad$ $V = \frac{M}{\rho} \,/\, \frac{1}{V} = \frac{\rho}{M}$

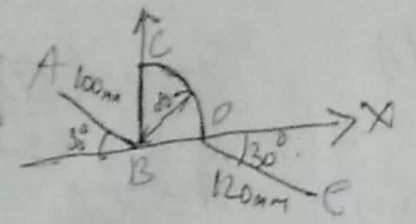
$= \frac{3}{\pi R^2 h} \frac{\pi R^2}{h^2} \left[\frac{hy^2}{2} - \frac{2hy^3}{3} + \frac{y^4}{4}\right]_0^h$ $\quad$ Volume of Cone $= \frac{\pi R^2 h}{3}$

$= \frac{3}{h^3}\left[\frac{3h^4}{4} - \frac{2h^4}{3}\right]$

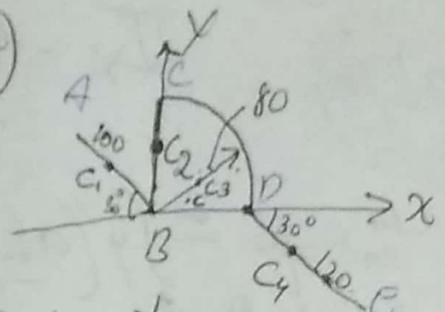
$= \frac{3}{h^3} \times \frac{h^4}{12} = \frac{h}{4}$, So centre of gravity of right circular solid cone is at distance of $\frac{h}{4}$ from its base along vertical axis

$$\boxed{y_{C.G} = \frac{h}{4}}$$

* Locate the centroid of wire bent as shown in fig.



A)



A Let us divide the bent wire into various segments with centroids C_1, C_2, C_3 and C_4.

Taking point B as origin, for segment CD (quarter circle) the coordinates of its centroid: $\bar{x} = \frac{2R}{\pi} = \frac{2 \times 80}{\pi} = 50.93 = \bar{y}$

Length $CD = \frac{\pi}{2} \times 80 = 125.66$

Segment	Length (l)	$\bar{x}$ (mm)	$\bar{y}$	$l\bar{x}$	$l\bar{y}$
AB	100	$-50 \cos 30 = -43.3$	$50 \sin 30° = 25$	-4330	2500
BC	80	0	$\frac{80}{2} = 40$	0	3200
CD	125.66	50.93	50.93	6399.86	6399.86
DE	120	$\frac{120 \cos 30}{2} + 80 = 131.96$	$-\frac{120 \sin 30}{2} = -30$	15835.20	-3600
	425.66			17905.06	8499.86

$\bar{X} = \frac{\Sigma l\bar{x}}{\Sigma l} = \frac{17905.06}{425.66} = 42.06$ mm

$\bar{Y} = \frac{8499.86}{425.66} = 20$ mm

* Differentiate between polar moment of inertia and product of inertia ?

A).

Polar moment of Inertia	Product of Inertia
1) Polar moment of Inertia is quantity used to describe resistance to torsional deformation in Cylindrical objects $I = \cancel{\int r^2 dm} \int_A r^2 dA$	1). Product of Inertia is defined as I_{xy}, I_{yz}, I_{xz}. According to Dynamics, Product of Inertia & boards assembly such that it will flip the assemble outside board which leads to dynamic imbalance $I_{xy} = \int_A xy \, dA$
2) For Symmetric shapes, Polar moment of Inertia has some value.	2) For Symmetric shapes product of inertia is zero.

* Determine the area generated by rotating a line of length 'l' about x-axis from a distance 'r' using Pappus theorem

A) <u>Theorem</u>

The area of surface of revolution is equal to the product of length of generating curve and distance travelled by the centroid of curve while the surface is being generated.

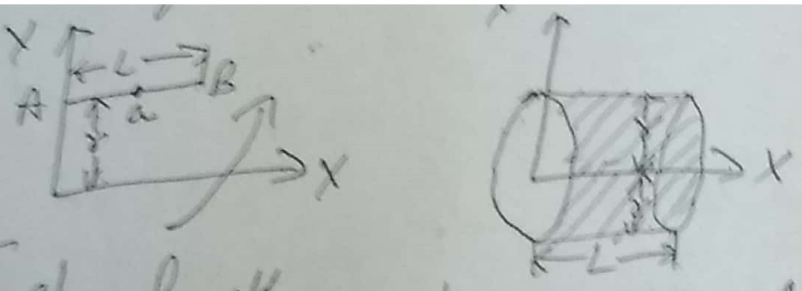
The generating curve must not cross the axis about which it is rotated

Length of generative curve $= L$

Distance travelled by centroid while the surface is being generated $= 2\pi r$ (Circumference of a circle of radius r)

$\therefore$ Area of surface of cylinder generated $= L \times 2\pi r$

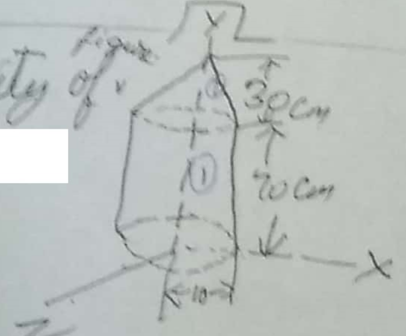
$= 2\pi r L =$ Area generated by rotating a line



In above fig, the curved surface of cylinder is obtained by rotating line AB about the x-axis

* Determine the centre of gravity of figure

A)



Volume of cylinder $= \pi R^2 h$

$= \pi \times 5^2 \times 40$

$= 3141.59\ cm^3$

Volume of cone $= \frac{1}{3}\pi \times 5^2 \times 30 = 785.39$

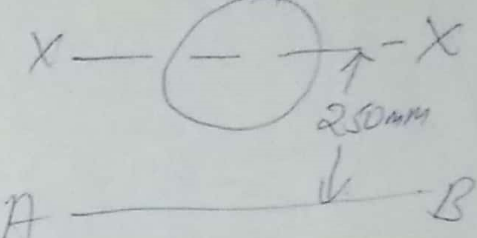
$\bar{x} = \frac{10}{2} = 5$ | $y_1 = \frac{40}{2} = 20$ | $y_2 = 40 + \frac{30}{4} = 47.5$ $\bar{z} = \frac{10}{2} = 5cm$

C.G of cone $= \frac{h}{4} = \frac{30}{4} = 7.5$ $\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} = 25.5cm$

* A Circle of dia 120mm is placed above on axis AB in such a way that is centre is 250mm above X axis AB as shown in fig Using parallel axis theorem, determine moment of inertia about reference axis AB and also determine the polar moment of inertia.

A)

$I_{AB} = I_{xx} + Ah^2$



$I_{xx} = \frac{\pi d^4}{64} = \frac{\pi (120)^4}{64} = 1.017 \times 10^{-5} m^4$

$h = 250mm$

$h = 0.25m$

J = Polar moment of Inertia

Area of Circle $= \frac{\pi (0.12)^2}{4} = 0.011 m^2$

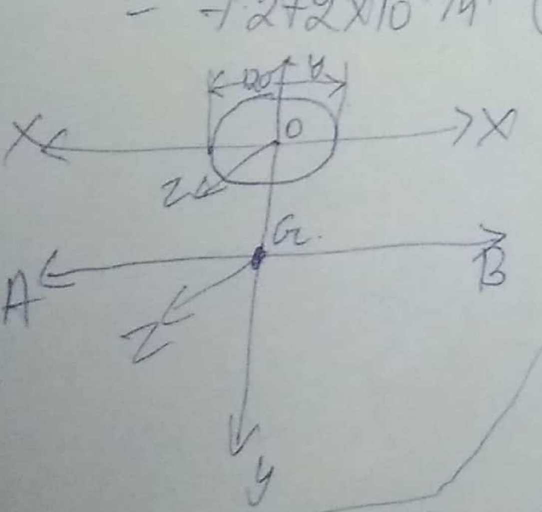
$I_{AB} = 1.017 \times 10^{-5} + (0.011)(0.25)^2 = 6.97 \times 10^{-4} m^4$

$I_{zz} = J_p = \frac{\pi (0.12)^4}{32} = 2.035 \times 10^{-5} m^4 = I_{xx} + I_{xy}$

$J_{AB} = (I_{xx} + I_{yy}) + Ah^2 = 2.035 \times 10^{-5} + \frac{\pi (0.12)^2}{4} \times (0.25)^2$

$= 7.272 \times 10^{-4} m^4$ (J_{AB} - Polar M.I about Centroid)

J = I_{zz} = Polar M.I about centre O

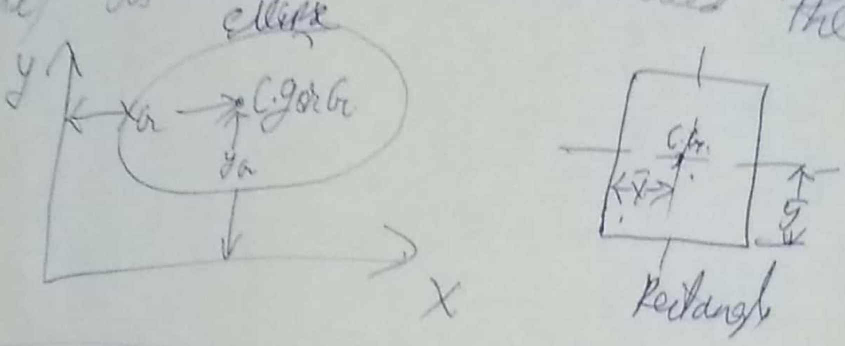


* Define Centroid and Centre of gravity with Examples?

A) The Centre of gravity of a body is the point, through which the whole weight of body acts, irrespective

* of the position in which body is placed. This can also be defined as centre of gravitational forces acting on a body. It is denoted by G or CG

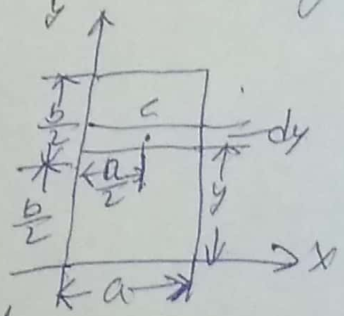
Centroid: It is defined as the point at which the total area of plane figure (like rectangle, square, triangle etc) is assumed to be concentrated. The centroid.



* Find the product of inertia of a rectangle of sides a and b with respect to the axes that lie along its two sides.?

A) a = width, b = height

$\bar{x} = \frac{a}{2}$, $y = y$



Consider a differential element of thickness dy and its height from x-axis

Area of differential element, $dA = a\,dy$

Product of inertia of element about x-axes & y axes

$$dI_{xy} = xy\,dA = \frac{a}{2} \times y \times a\,dy = \frac{a^2}{2} y\,dy$$

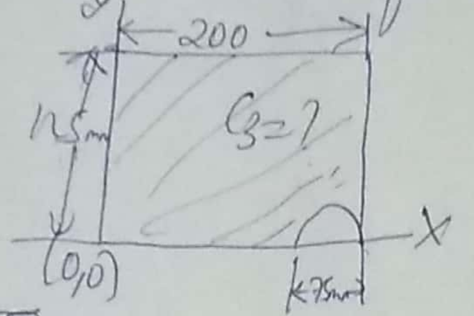
Product of inertia of entire rectangular area is

$$I_{xy} = \int dI_{xy} = \int_0^b \frac{a^2}{2} y\,dy = \frac{a^2}{2}\frac{b^2}{2} = \frac{a^2 b^2}{4}$$

Altre

$$I_{xy} = \int_A xy\,dA = \int_0^a\int_0^b yx\,dx\,dy = \frac{b^2a^2}{4}$$

* Locate the centroid for the shaded area as in figure



A) Rectangle area $= 200 \times 125$

$A_1 = 25000\,mm^2$
$= 0.025\,m^2$

$x_1 = 100 = 0.1\,m$
$y_1 = 62.5 = 0.0625\,m$

Semicircular area $A_2 = \frac{\pi R^2}{2} = \frac{\pi \times 37.5^2}{2} = 2.208 \times 10^{-3}\,m^2$

$x_2 = 200 - \frac{75}{2} = 162.5\,mm = 0.1625\,m$

$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$

$y_2 = \frac{4r}{3\pi} = \frac{4 \times 37.5}{3\pi} = 15.91\,mm = 0.0159\,m$

$\bar{y} = 0.067\,m$
$= 67\,mm.$

$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = 0.093\,m$
$= 93.94\,mm.$

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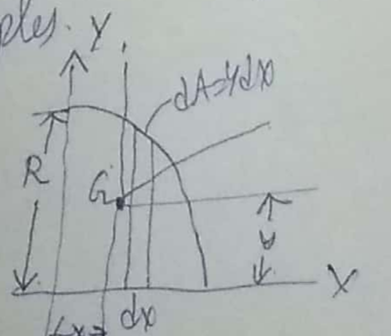
* Find centroid of quarter circular line from basic principles.

A)



$x_G = \frac{\int_A x\,dA}{\int_A dA}$

in this $dA = x\,dy$

$x^2 + y^2 = R^2.$

$y_G = \frac{\int_A y\,dA}{\int_A dA}$

In this $dA = y\,dx$

$$y_G = \frac{\int_0 \frac{y}{2} x y\, dx}{\frac{\pi R^2}{4}} = \frac{2\int (R^2-x^2)\, dx}{\pi R^2}$$

$$= \frac{2\left[R^2 x - \frac{x^3}{2}\right]_0^R}{\pi R^2}$$

$$= \frac{2\left[R^3 - \frac{R^3}{2}\right]_0^R}{\pi R^2}$$

$$y_G = \frac{4R}{3\pi}$$

$$X_G = \frac{\int_0^R \frac{x}{2}(x)\, dy}{\frac{\pi R^2}{4}}$$

$$= \frac{2\int (R^2-y^2)\, dy}{\pi R^2}$$

$$= \frac{2\left[R^2 y - \frac{y^3}{2}\right]_0^R}{\pi R^2}$$

$$X_G = \frac{2\left[R^3 - \frac{R^3}{2}\right]_0^R}{\pi R^2} = \frac{4R}{3\pi}$$

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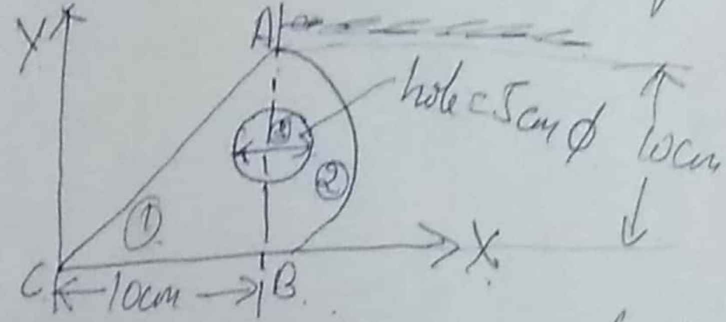
*. Determine the moment of inertia for the area about axis AB.



A) $A_1 = \frac{1}{2} \times 10 \times 10 = 50cm^2 = 0.005 m^2$

$A_2 = \frac{\pi \times 5^2}{2} = 39.26 cm^2 = 3.926 \times 10^{-3} m^2$

$A_3 = -\pi \times 2.5^2 = -19.634 = -1.96 \times 10^{-3} m^2$

Distance of centroid from axis Y axis: $X_1 = \frac{10}{3} = 3.33 cm = 0.03 m$, $X_2 = -\frac{4 \times 5}{3\pi} = -2.122 cm$

$X_3 = 0$.

Taking moments of these area about axis Y and equating their sum to moment of entire area about AB axes, we get, $\bar{X} = \frac{0.005(0.03) + 3.926\times10^{-3}(-2.122 \times 10^{-2}) - (1.96\times10^{-3})(0)}{0.005 + 3.926 \times 10^{-3} - 1.96 \times 10^{-3}}$

$\bar{X} = 1.19 cm = 0.012 cm$

To determine moment of Inertia at axis AB

Component	Area A (cm^2)	M.I about (I_c) Centroidal axis I_c	Distance b/w AB & Y axis x (cm)	Ax^2	M.I about AB axis $I_{AB} = I_c + Ay^2$
Triangle	50	$\frac{bh^3}{36} = \frac{10 \times 10^3}{36} = 277.77$	$\bar{x}_1 - \bar{x}$ $= 3.33 - 1.2$ $= 2.13$	226.87	504.615
Semi-circle	39.26	$0.11r^4 = 68.75$ $r = 5cm$	$x_2 - \bar{x}$ $= -2.12 - 1.2$ $= -3.32$	432.73	501.48
Circular hole	-19.634	$-\frac{\pi r^4}{4} = -30.67$ Here $r = 2.5cm$	$x_3 - \bar{x}$ $= 0 - 1.2$ $= -1.2$	-28.27	-58.94

$I_{AB} = 947.155 \, cm^4$

* State and prove Pappus Theorem of Volume & Area.

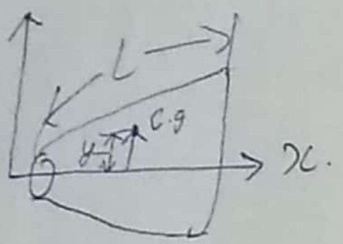
A) Pappus theorems are useful in computing the surface area or Volume of body generated by revolution

Theorem 1 (Area):- The surface area generated by revolving a planar curve about a non-intersecting axis in the plane of curve is equal to the product of length of curve and distance traversed by the centroid of the curve

Proof:- Consider a curve of length L in x-y plane, which is revolved by 360° about x-axis to create a surface

Surface area A. The area dA generated by a small segment dL of curve is given by, $dA = 2\pi y \, dL$, where y is the mean radius of circular path generated by dL.

The Total area A is obtained by integrating dA over length of the curve as $A = \int_L dL = \int_L 2\pi y \, dL = 2\pi \int_L y \, dL$.



But $\int_L y \, dL$ is the total moment of length L, about x-axis and from

$$\int_L y \, dL = \bar{y} L.$$

$$\text{So } A = \bar{y} L (2\pi)$$

$2\pi\bar{y}$ is distance travelled by Centroid C of the curve.

(Area) of (A) = Length of curve (L) × Distance travelled by Centroid ($2\pi\bar{y}$)
Generated Surface

Pappus Theorem-2

The Volume generated by revolving a plane of area A about a non-intersecting axis in the plane of area is equal to product of area of plane and distance travelled by centroid of area

Proof: Consider a plane of area A in x-y plane, which is revolved by 360° about X-axis to Create a volume V. The volume dV generated by a Small element of area dA of plane is given by $dV = 2\pi y \, dA$, where y is mean radius of circular path generated by dA. Total volume generated is obtained by integrating dV over area of plane as

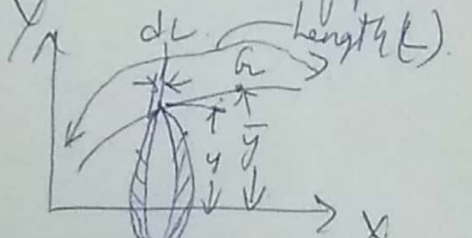
$$V = \int dV = \int_A 2\pi y\,dA = 2\pi \int_A y\,dA$$

But $\int_A y\,dA$ is total moment of area, A about X-axis, and from we have $\int_A y\,dA = \bar{y}A$

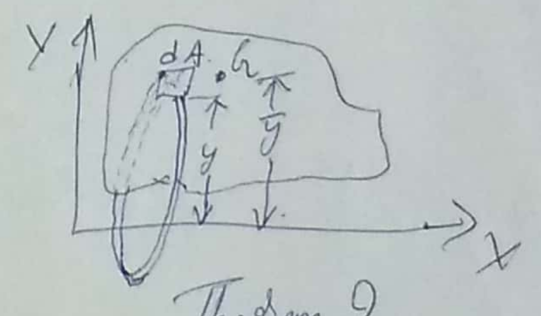
$$V = 2\pi \int_A y\,dA = 2\pi \bar{y}\,dA$$

$2\pi\bar{y}$ is distance traced by centroid C of plane. Therefore volume V of generated body is given as

V = Area of plane X Distance traced by centroid.



Theorem 1



Theorem 2

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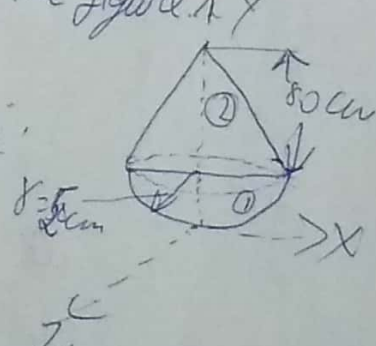
* Determine the centre of gravity of the figure

A) As the body is symmetrical about Y-Y axis therefore its C.g will lie on Y-Y axis



① Hemisphere

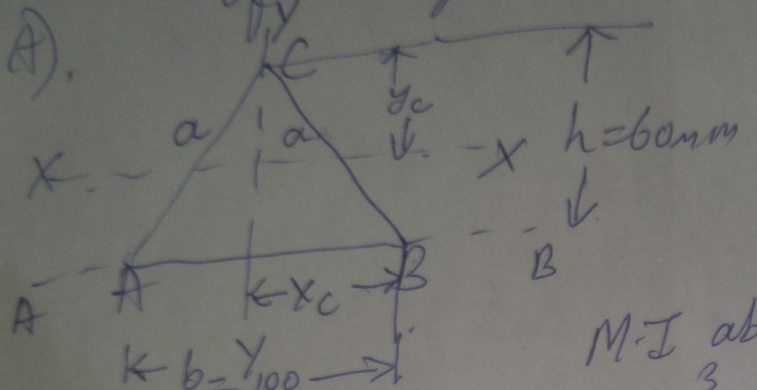
$V_1 = \frac{2\pi}{3} \times r^3 = 261799.38\ cm^3$ | $y_1 = \frac{5 \times 50}{8} = 31.25\ cm$

② Right circular cone, $V_2 = \frac{\pi r^2 h}{3} = \frac{\pi}{3} \times (50)^2 \times 80 = 209439.51\ cm^3$ | $y_2 = 50 + \frac{80}{4} = 70\ cm$

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} = 48.47\ cm$$

Q) An isosceles Δle section ABC has a base of 100mm and 60mm height. Determine the moment of inertia of triangle about centroid and about base

A).



M.I about Centroid

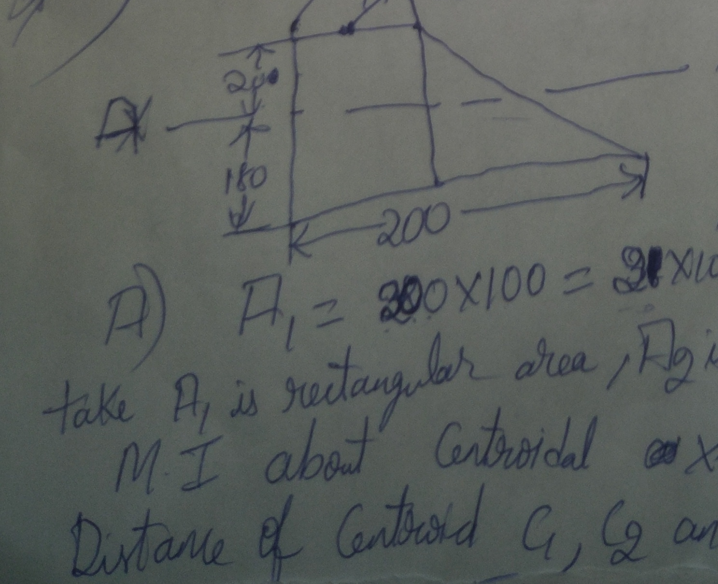
$I_{xx} = \frac{bh^3}{36} = \frac{100\times60^3}{36} = 600000\ mm^4$

$I_{yy} = \frac{b^3h}{48} = \frac{100^3\times60}{48} = 1250000\ mm^4$

M.I about base

$I_{AB} = \frac{bh^3}{12} = 1800000\ mm^4$

Find the moment of inertia above given AB axes



A) $A_1 = 200\times100 = 2\times10^4\ mm^2$ | $A_2 = \frac{1}{2}\times200\times100 = 1\times10^4\ mm^2$

take A_1 is rectangular area, A_2 is Triangle area, A_3 is Semicircle area.

$A_3 = \frac{\pi\times50^2}{2} = 3926.99$

M.I about Centroidal X-axis.

Distance of Centroid C_1, C_2 and C_3 from X-axis

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$= 109.1163.$

$y_1 = \frac{200}{2} = 100$

$y_2 = \frac{200}{3} = 66.66.$

$y_3 = 200 + \frac{4 \times 50}{\pi}$

$= 263.66$

Distance of Centroids C_1, C_2 and C_3 from AB axis passing through centroid at AB axis. Keep $y = 180$

$-\bar{y} + y = 180 - 109.1163 = 70.89 = h$

M.I of AB axes $= \left(\frac{bd^3}{12} + Ah^2\right)_{\text{rectangle}} + \left(\frac{bd^3}{36} + Ah^2\right)_{\text{triangle}}$

$b = 100,\ d = 200$

$A = 2 \times 10^4\ mm^2,\ h = 70.89$

$b = 100$

$d = 200.$

$A = 1 \times 10^4\ mm^2$

$h = 70.89$

$+ \left(0.11 \times (50^4) + Ah^2\right)$

$A = 3926.99,\ h = 70.89$

M.I of AB axes $= (167174508.7)_{\square le} + (72476143.22)_{\Delta le}$

$+ (20422164.52)_{\triangle le} = 260072816.5\ mm^4$

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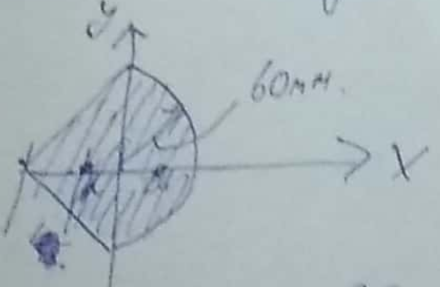
* Locate the centroid of shaded area



A)

Segment	Area	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
Semi Circle	$\left(\frac{\pi \times R^2}{2}\right)$	$\frac{4R}{3\pi}$	0	$\frac{2R^3}{3}$	0
triangle	$\frac{bh}{2}$	$-\frac{h}{3}$	0	$-\frac{bh^2}{6}$	0

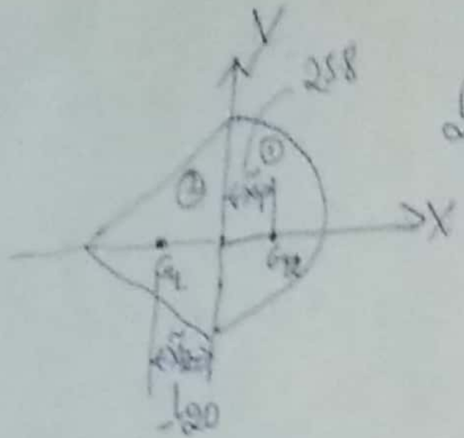
A) $\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{A\bar{x}_{\text{Semi Circle}} + A\bar{x}_{\Delta le}}{A_{\text{Semi Circle}} + A_{\Delta le}} = 7.84$

$b = 60 + 60 = 120.$

$h = 60.$

$\bar{x}_{\text{Semi circle}} = 25.46$

$\bar{x}_{\Delta le} = -20$



Semi circle $\left(\bar{X} = \frac{4R}{3\pi} = 25.8\right)$ $\left(\bar{x}_{\Delta le} = \frac{-h}{3} = \frac{-60}{3} = -20\right)$

$\bar{X} = 78\,mm$

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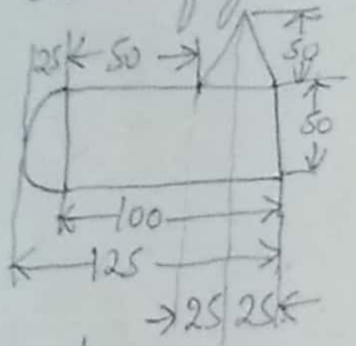
* Uniform lamina consists of rectangle, a semicircle and a Δle. Find Centre of gravity.



A) Rectangular portion

$a_1 = 100 \times 50 = 5000\,mm^2$

$x_1 = 25 + \frac{100}{2} = 75\,mm.$

$y_1 = \frac{50}{2} = 25\,mm$

Semicircular portion

$a_2 = \frac{\pi r^2}{2} = \frac{\pi}{2} \times 25^2 = 982\,mm^2$, $x_2 = 25 - \frac{4r}{3\pi} = 25 - \frac{4 \times 25}{3\pi} = 14.4\,mm$

$y_2 = \frac{50}{2} = 25\,mm$

Triangular portion $a_3 = \frac{50 \times 50}{2} = 1250\,mm^2$, $x_3 = 25 + 50 + 25 = 100\,mm$

$y_3 = 50 + \frac{50}{3} = 66.7\,mm$

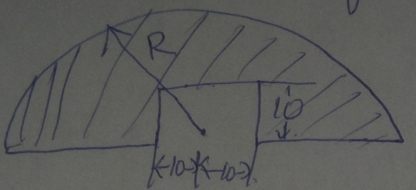
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250}$$

$\bar{x} = 71.1\,mm$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250}$$

$\bar{y} = 32.2\,mm.$

* Locate the centroid of hatched area, $r = 30$ mm



A) $A_1 = \frac{\pi \times 30^2}{2} = 1413.71\ mm^2$ ~~$x_1 = \frac{60}{2} = 30$~~ | ~~$y_1 = \frac{4 \times 30}{3\pi} = 12.73$~~

$A_2 = 10 \times 20 = 200.$ ~~$x_2 = \frac{10}{2} = 5$~~ | ~~$y_2 = \frac{10}{2} = 5$~~

Symmetry of Y axis ($\bar{X} = 30$)

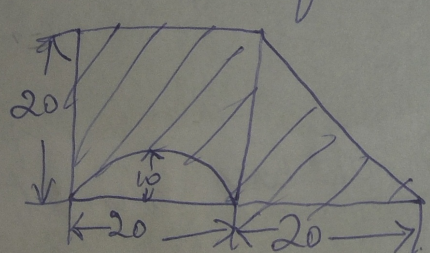
$y_1 = \frac{4 \times 30}{3\pi} = 12.73$

$y_2 = \frac{10}{2} = 5.$

$$\bar{Y} = \frac{1413.71 \times 12.73 - 200 \times 5}{1413.71 - 200}$$

$\bar{Y} = 14.$

* Find M.I for hatched area. area parallel to x-axis



A) Area of square $= 20 \times 20 = 400$, $x_1 = \frac{20}{2} = 10$ | $y_1 = \frac{20}{2} = 10$

Area of $\Delta le = \frac{1}{2} \times 20 \times 20 = 200$, $x_2 = 20 + \frac{20}{3} = 26.66$ | $y_2 = \frac{20}{3}$

Area of semicircle $= \frac{\pi R^2}{2} = 157.07$ | $x_3 = \frac{20}{10} = 10$ | $y_3 = \frac{4 \times 10}{3\pi} = 4.24$ m

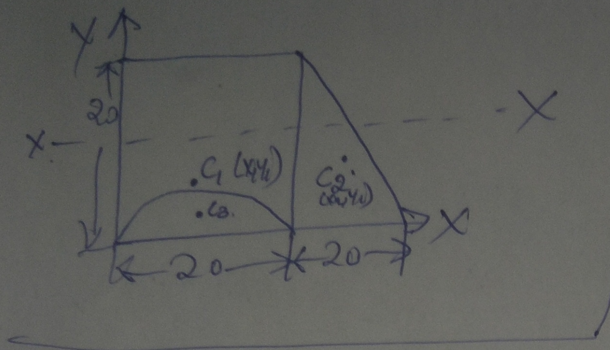
$\bar{x} = 17.52.$ $\bar{y} = 10.53$ | $y_1 - \bar{y} = 10 - 10.53 = 0.53$ | $y_2 = \frac{20}{3} = 6.66$ m.

$\bar{y}_1 = y_1 - \bar{y} = 0.53$ | $\bar{y}_2 = \bar{y} - y_2 = 10.53 - 6.66 = 3.863$

$\bar{y}_3 = \bar{y} - y_3 = 10.53 - 4.24 = 6.3$

$$I_{xx} = \left(\frac{20^4}{12} + (20)(20)(0.53)^2\right)_{\text{Square}} + \left(\frac{20^4}{36} + 200(3.86)^2\right)_{\Delta le}$$

$$- \left(0.11 \times 10^4 + 157.07(6.29^2)\right) = 13555.72\ m^4$$



* Write the numerical formula of Polar M.I.

A) $J = \int dJ$

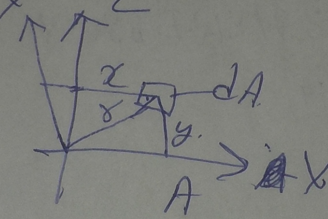
Polar M.I of entire area $= \int r^2 dA$

$r^2 = x^2 + y^2$

$J = \int (x^2 + y^2) = I_y + I_x$

M.I of an ~~axis~~ area about an axis $\perp$ to its plane is called Polar M.I.

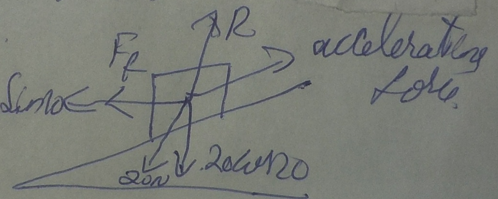
Lamina of area A in xy plane



* A body weighing 20N is projected up a 20° inclined plane with a velocity of 12m/s. The coefficient of friction is 0.15. Find distance travelled

A). $F_g = Mg \sin 20$ | Take $Mg = 20N$ |

$F_f = +0.15(mg \cos 20)$



-ve sign becoz due to deceleration

Total decelerating force $= -(mg(\sin 20 + 0.15 \cos 20°))$

$= -m(4.728)$

Total accelerating force = Total decelerating force

$ma = -m(4.728)$, $a = -4.728$

$v^2 - u^2 = 2as$ (Constant acceleration)

$0^2 - 12^2 = 2(-4.728)(s)$ $s = 15.2m$.

* State and prove the perpendicular axis theorem with suitable diagram

A: Perpendicular axis states: If I_{OX} and I_{OY} be the M.I of a lamina about mutually ⊥ axes OX and OY in plane of lamina and I_{OZ} be the M.I of lamina about an axis (OZ) normal to the lamina and passing through the point of intersection of the axes OX and OY, then $I_{OZ} = I_{OX} + I_{OY}$

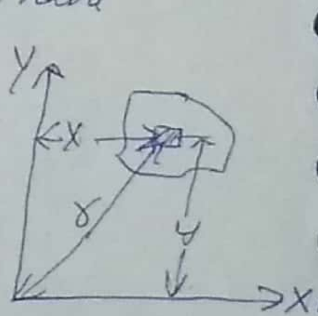
Let OX and OY be the two mutually ⊥ axes lying in plane of lamina. Let OZ be axis normal to the lamina and passing through O.

Consider an elemental component of area dA of the lamina. Let the distance of this elemental component from the axis OZ i.e from O to be r.

Then M.I of elemental component about $OZ = dA \times r^2$

If the co-ordinates of elemental components be (x, y) referred to axes OX and OY, we have

$r^2 = x^2 + y^2$



M.I of elemental component about axis OZ

$= dA \cdot (x^2 + y^2) = dAx^2 + dAy^2$

∴ Total M.I of lamina about axis OZ

$= I_{OZ} = \Sigma (dAx^2 + dAy^2) = \Sigma dAx^2 + \Sigma dAy^2$

But ΣdAx^2 = M.I of lamina about axis $OY = I_{OY}$

ΣdAy^2 = M.I of lamina about axis $OX = I_{OX}$

Hence $I_{OZ} = I_{OX} + I_{OY}$

* What is Significance of Moment of Inertia?

A). The moment of inertia of a rigid body about a given axis is, defined as the sum of products of mass of each and every particle of the body and square of its distance from the given axis

S.I units - $Kg\,m^2$. Significance :-

1) In the translational motion, the mass of a body is a measure of its inertia. Greater the mass, larger is the inertia, greater is force required to produce a given linear acceleration is it.

2) In rotational motion, the moment of inertia of a body is a measure of its inertia. Greater the moment of inertia, larger is the torque required to produce a given angular acceleration in it

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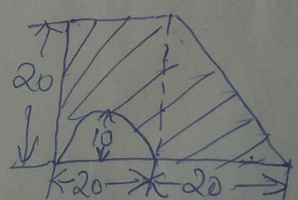
* Locate the centroid of hatched area



A) Area of Square $= 20 \times 20 = 400\,cm^2$.

$x_1 = \frac{20}{2} = 10cm$ | $y_1 = \frac{20}{2} = 10cm$.

Area of triangle $= 20 \times 20 \times \frac{1}{2} = 200\,cm^2$.

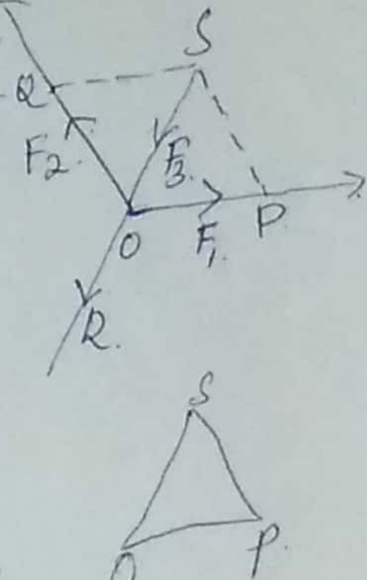
$x_2 = 20 + \frac{20}{3} = 26.667\,cm$ | $y_2 = \frac{20}{3} = 6.667$

Area of Semi circle $= 157.07\,cm^2 = \pi R^2/2$.

$x_3 = \frac{20}{2} = 10m$ $y_3 = \frac{4 \times 10}{3\pi} = 4.24\,m$

$\bar{x} = 17.52$, $\bar{y} = 10.53$

* ~~Two blak~~. State the converse of law of triangle of forces.



A) If three forces acting at a point are in equilibrium, then they can be represented in magnitude as well as direction by the three sides of triangle taken in order such that its sides are parallel to the direction of the forces respectively

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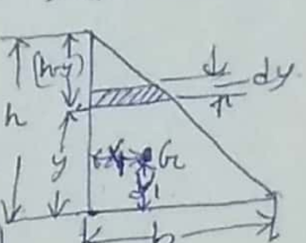
* From the first principle, find the centroid of a right angle triangle of height 'h' and breadth 'b'.



A) Consider a right angle Δle with a base (b) and height (h). Let (G) be the centroid of Δle. Let us consider the X-axis and Y-axis. Let us consider an elemental area (dA) of width (b_1) and thickness dy, lying at a distance (y) from X-axis.

$$\bar{y} = \int_0^h \frac{y\,dA}{A} \quad , \quad A = \frac{bh}{2} \quad , \quad dA = b_1 dy \quad \Big| \quad \bar{Y} = \frac{\int_0^h y(b_1 dy)}{\frac{bh}{2}}$$

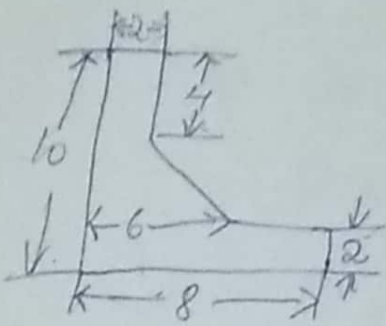
(as x varies, b_1 also varies)

$$\left(\frac{h-y}{h} = \frac{b_1}{b}\right)$$

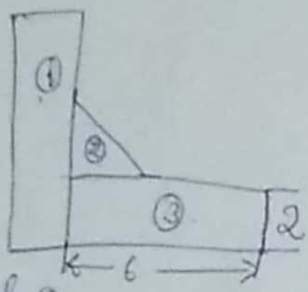
$$\bar{y} = \frac{2}{h} \int_0^h \left(y - \frac{y^2}{h}\right) dy$$

$$\bar{y} = \frac{2h}{6} \quad , \quad \bar{Y} = \frac{h}{3} \quad , \quad \bar{X} = \frac{h}{3}$$

* Find the Centroid of area



A)



Rectangle ①

$a_1 = 10 \times 2 = 20\,mm^2$

$x_1 = \frac{2}{2} = 1\,mm$, $y_1 = \frac{10}{2} = 5$

Δle

$a_2 = \frac{1}{2} \times 4 \times 4 = 8$ (4, $4\sqrt{2}$, 4)

$x_2 = 2 + \frac{4}{3} = \frac{10}{3}\,mm$

$y_2 = 2 + \frac{4}{3} = \frac{10}{3}\,mm$

Rectangle ③

$a_3 = 6 \times 2 = 12\,mm^2$ / $x_3 = 2 + 4 + \frac{2}{2} = 7$ / $y_3 = \frac{2}{2} = 1$.

$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = 3.266$ / $\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$

$\bar{y} = 3.8$

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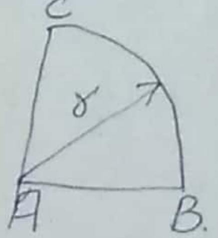
* Derive M.I of a quarter circle of radius (r) about the base and the centroidal axis.

A) Consider a quarter circle ABC



$I_{AB} = \frac{1}{4} \times \frac{\pi r^4}{4} = 0.196 r^4$

Area of Circular section, $A = \frac{\pi r^2}{4}$ / $h = \frac{4r}{3\pi}$

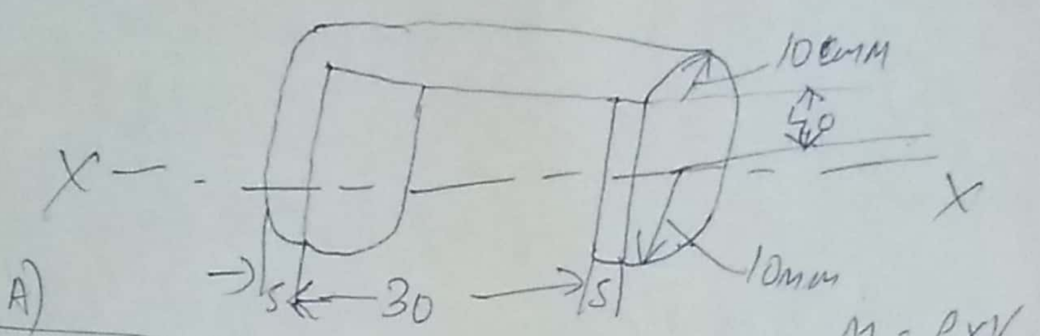
Distance b/w. C.g and base AB

$I_{G\,(\text{Centroid})} = I_{AB} - ah^2 = \left(\frac{\pi r^4}{16}\right) - \left(\frac{\pi r^2}{4}\right)\left(\frac{4r}{3\pi}\right)^2$

$= 0.0548\, r^4$.

$I_{AB\,(\text{base})} = 0.196 r^4$

* Compute the mass M.I about the (X-axis) of steel link



A)

$M = \rho \times V$

Component	Mass	Mass M.I about Centroidal axis, I_c	Distance b/w Centroidal axis and axis, y	my^2	Mass M.I about x-axis $I_x = I_c + my^2$
Half cylinder (10mm radius, 40mm length)	$\frac{\rho\pi r^2 h}{2}$ $= \frac{\rho\pi(10)^2 \times 40}{2}$ $= 6283\rho$	$0.32 m r^2$ $= 0.32 \times 6283 \times \rho \times 10^2$	$40 + \frac{4r}{3\pi}$ $= \frac{40 + 4(10)}{3\pi} = 44.24$	1222-694ρ	12498002.8ρ
Two rectangle prism of dimensions (40×20×5)	$(40 \times 20 \times 5\rho)$ $= 4000\rho$	$\frac{m(40^2 + 20^2)}{12}$ $= 666666.67\rho$	20	1600000ρ	2266666.67ρ × 2 $= 4533333.33\rho$
Two half discs (each of 10mm radius and 5mm thickness)	$\frac{\rho\pi r^2 h}{2}$ $= \frac{\rho\pi(10)^2 \times 5}{2}$ $= 785.38\rho$	$0.32 m r^2$ $= 0.32 \times 785.38\rho \times 10^2$ $= 25132.16\rho$	$\frac{4r}{3\pi}$ $= \frac{4(10)}{3\pi}$ $= 4.24$	14119.29ρ	39251.4ρ×2 $= 78502.82\rho$

$M = \Sigma M = (6283 + 24000 + 2\times(785.38))\rho = 15853.76\rho$

$\Sigma I_x = 17109.839\rho$

$I_x = 17109.839\rho \;/\; \rho = \frac{M}{15853.76}$

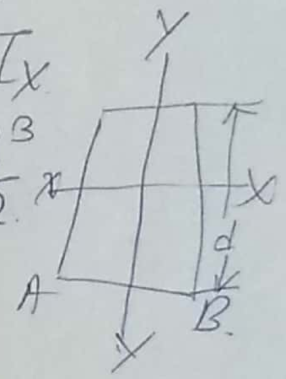
$I_x = 17109.839 \times \frac{M}{15853.76} = (1079.23M)\, gm\, m^2$

* Determine the radius of gyration for rectangle about ① x-axis ⓑ its base

A) Radius of gyration is distance from an axis on which the entire area (or) mass of a body may be Squeezed (Compress) in form of a thin strip (or a point) to yield the same area M.I about the axis.

The distance (K_y) is called radius of gyration of area (A) about x-axis. Similary K_x about y-axis

& M.I I_x about x-axis., $AK_y^2 = I_x$



$K_y = \sqrt{\frac{I_x}{A}}$ $\quad I_{xx}$ of rectangle $= \frac{bd^3}{12}$

$K_y = \sqrt{\frac{\frac{bd^3}{12}}{bd}} = 0.288(\sqrt{d})^2$ $\quad$ Area $= bd^2$

$= 0.288\, d$

K_y about x-axis $= 0.288\sqrt{d}$.

Radius of gyration for □le about its base.

$I_{AB} = I_{xx} + Ah^2 = \frac{bd^3}{12} + db\left(\frac{d}{2}\right)^2 \quad h = d/2$

$I_{AB} = \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{d^3 b}{3}$

$K_{base} = \sqrt{\frac{\frac{bd^3}{3}}{bd}} = 0.577d$

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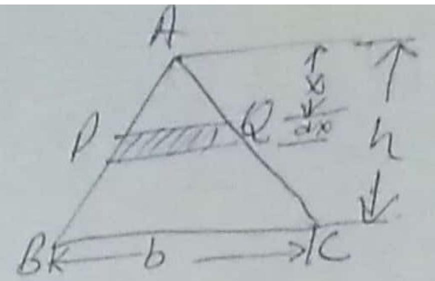
* Statarting from first principles determine M.I of a Δle w.r.to its base.

A). Consider a Δle ABC



Let b = Base of Δle

h = height of Δle

Now Consider a small strip (PQ) of thickness (dx) at a distance of x from the vertex A. From the geometry of the fig we find the two Δle APQ and ABC are similar,

$$\therefore \frac{PQ}{BC} = \frac{x}{h} \quad \Big| \quad PQ = \frac{BC(x)}{h} = \frac{bx}{h}$$

area of strip $PQ = \frac{bx}{h} dx$

M.I of strip about base BC

$$= \text{Area} \times (\text{Distance})^2 = \frac{bx}{h} dx (h-x)^2 = \frac{bx}{h} (h-x)^2 dx$$

M.I of whole 'Δle' is find out by Integrating Equation for whole height of Δle from 0 to h

$$I_{BC} = \int_0^h \frac{bx}{h} (h-x)^2 dx$$

$$= \frac{b}{h} \int_0^h (xh^2 + x^3 - 2hx^2) dx = \frac{b}{h} \left[\frac{x^2h^2}{2} + \frac{x^4}{4} - \frac{2hx^3}{3} \right]_0^h$$

$$= \frac{bh^3}{12}$$ If we take h = depth = d, then $I_{BC} = \frac{bd^3}{12}$

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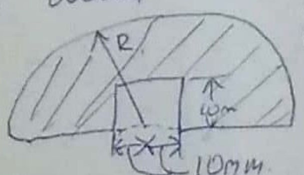
* Find the M.I about horizontal centroidal axis of shaded portion



R=30mm

A) $A_1 = \frac{\pi (30)^2}{2} = 1413.71 mm^2$ $\quad$ / $A_2 = 10 \times 20 = 200$

Symmetry of Y axis ($\bar{x} = 30$)

$$y_1 = \frac{4 \times 30}{3\pi} = 12.73$$

$$y_2 = \frac{10}{2} = 5$$

$$\bar{Y} = \frac{(1413.71)\times(12.73) - (200 \times 5)}{1413.71 - 200} = 14$$

$I_{\cancel{xx}\,CG} = (0.11r^4) - \left(\frac{bd^3}{12}\right)$

Area of semi circle   Area of □le.

$r = 30m,\ b = 20,\ d = 10$

$I_a = 87433.33\ mm^4$

M.I of horizontal Centroidal axes

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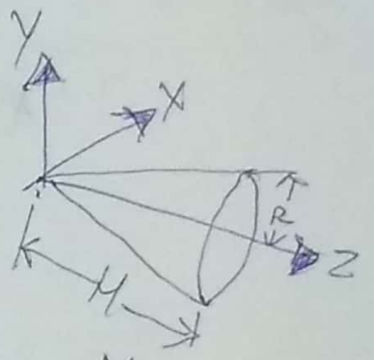
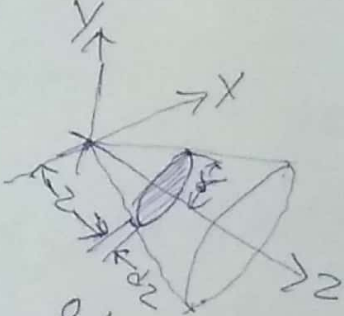
* Deduce (Find) an equation for M.I of right circular solid cone about its generation axis of base (R) and mass M about axis of cone.



A). Consider a cone of base radius R, height H and mass density ρ, oriented with respect to (w.r.t) to axes.

Suppose we cut a circular disc of radius r and infinitesimal thickness (dz) at a distance (z) from origin.



Section of cone

$$dm = \rho \pi r^2 dz.$$

$\therefore$ Mass M.I $(dI_{zz})_{mass} = \frac{dm\, r^2}{2}$ | $K = \frac{r}{\sqrt{2}},\ K^2 = \frac{r^2}{2}$

$$= \rho\left(\frac{\pi r^4}{2}\right) dz.$$

$I_{zz} = \int_0^H \rho\left(\frac{\pi r^4}{2}\right) dz$ | $\frac{r}{R} = \frac{z}{H}$ (Due to similar Δle)

$$= \int_0^H \frac{\rho\pi}{2}\frac{R^4}{H^4} z^4 dz = \frac{\rho\pi}{2} \times \frac{R^4}{H^4}\int_0^H z^4 dz = \frac{\rho\pi}{2} \times \frac{R^4}{H^4}\left(\frac{H^5}{5}\right)$$

$$= \frac{\rho\pi R^4 H}{10}$$

we know that Volume of Cone $V = \frac{1}{3}\pi R^2 H$

$$M = \rho V = \frac{1}{3}\pi R^2 H \rho \quad \Big| \quad 3M = \pi R^2 H \rho \quad \text{—(a)}$$

$$I_{zz} = \frac{1}{10}\rho \pi R^4 H = \frac{1}{10}(\rho \pi R^2 H) R^2 = \frac{3}{10} M R^2 \quad \text{(using eq (a))}$$

$$\boxed{I_{zz} = \frac{3}{10} M R^2}$$

(b) The mass M.I of disc about an axis lying on it's plane is given as

$$(dI_{x'x'})_{\substack{\text{mass}\\\text{of disc}}} = dm \frac{r^2}{4} \qquad \left(K = \frac{r}{2} \text{ for solid disc}, \; K^2 = \frac{r^2}{4}\right)$$

$$= \rho \pi r^2 dz \, r^2/4 = \rho \frac{\pi r^4}{4} dz$$

By transfer theorem, mass M.I of disc about X-axis is given as

$$(dI_{xx})_{mass} = \rho\left(\frac{\pi r^4}{4}\right)dz + \underbrace{(\rho \pi r^2 dz)}_{m' dz^2} z^2$$

$M = \rho \pi r^2 dz$ mass of disc ~~of~~ ~~element~~

On integration b/w limits, we get mass M.I of cone about X (or) Y axis at vertex is

$$I_{xx} = I_{yy} = \int_0^H \rho\left(\frac{\pi r^4}{4}\right)dz + \int_0^H \rho \pi r^2 z^2 dz$$

$$= \frac{\rho \pi}{4}\frac{R^4}{H^4}\int_0^H z^4 dz + \frac{\rho \pi R^2}{H^2}\int_0^H z^4 dz \quad \left(\frac{r}{R} = \frac{z}{H}\right)$$

$$= \frac{1}{20}\rho \pi R^4 H + \frac{1}{5}\rho \pi R^2 H^3 \qquad \boxed{\text{Mass of cone } = \frac{\rho}{3}\pi R^2 H}$$

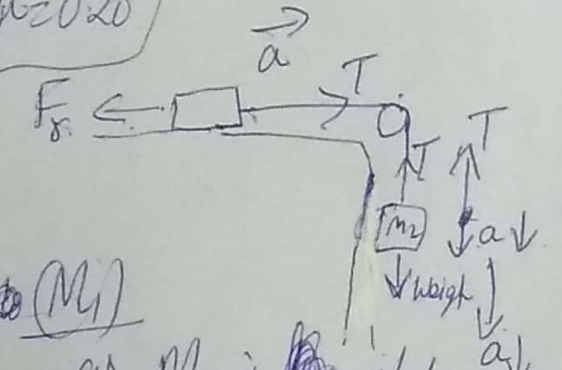
$$I_{xx} = I_{yy} = \frac{3}{5} M \left(\frac{R^2}{4} + H^2\right)$$

Note: If M.I of Cone about axes at base is required, it can obtained by applying parallel axis theorem to above value of I_{xx} or I_{yy}.

---

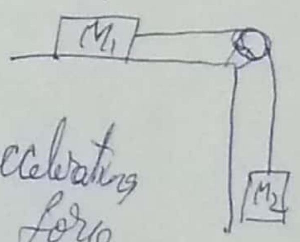
* Two blocks of masses (M_1) and (M_2) are connected by a string. Coefficient of friction b/w block M_1 and horizontal surface to be μ. If the system is released from rest, find velocity of block (A) after it has moved a distance of (m). Assume ($M_1 = 100kg$) ($M_2 = 150kg$) and ($\mu = 0.20$)

A)



$\Sigma F =$ accelerating force

T



(M_1)

as M_1 is sliding on horizontal surface, friction will act.

$$\Sigma F_H = M_1 a \qquad T = 100a + \mu mg$$

$$T - F_{friction} = M_1 a \qquad T = 100a + 0.2 \times 100 \times 9.81$$

$$T = 100a + 196.2 \quad —(1)$$

(M_2)

$$\Sigma F_V = M_2 a$$

$$T - \text{Weight} = 150 \times a$$

$$T - mg = 150a, \quad T = 150a + 150 \times 9.81$$

$$T = 150a + 1471.5 \quad (2)$$

$$100a + 196.2 = 150a + 1471.5 \quad (LHS = RHS)$$

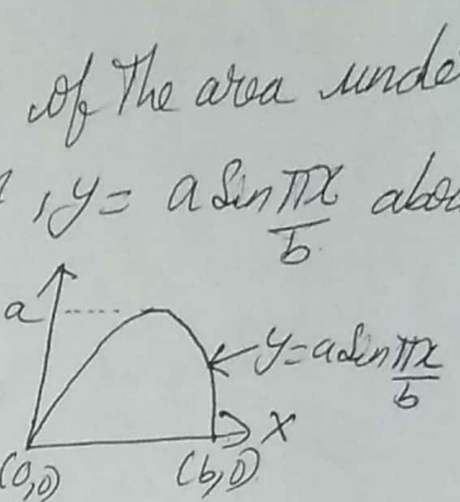
$$a = (-25.506)\, m/s^2$$

Velocity of

For constant acceleration
$v^2 - u^2 = \pm 2as$
final velocity = 0
$-u^2 = 2as$
$+u^2 = \pm 2 \times 25.5 \times 1$
$u_A = 7.14$ m/s

Determine the moment of inertia of the area under the sine curve with equation, $y = a\sin\frac{\pi x}{b}$ about X-axis

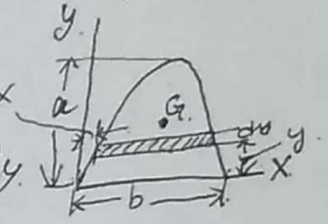
A) $I_x = \int_A y^2\,dA$



$$y = a\sin\left(\frac{\pi x}{b}\right)$$

$$\frac{y}{a} = \sin\left(\frac{\pi x}{b}\right) \quad \Big| \quad \sin^{-1}\left(\frac{y}{a}\right) = \frac{\pi x}{b} \quad \Big| \quad \frac{b}{\pi}\sin^{-1}\left(\frac{y}{a}\right) = x$$

$$dA = x\,dy = \frac{b}{\pi}\sin^{-1}\left(\frac{y}{a}\right)dy$$



$$I_x = \int_A y^2\,dA = \int_0^a y^2\left(\frac{b}{\pi}\sin^{-1}\left(\frac{y}{a}\right)\right)dy$$

$$= \frac{b}{\pi}\int_0^a y^2\sin^{-1}\left(\frac{y}{a}\right)dy = \frac{b}{\pi}\left[\sin^{-1}\left(\frac{y}{a}\right)\int_0^a y^2\,dy - \int_0^a\left[\frac{d}{dy}\sin^{-1}\left(\frac{y}{a}\right)\int y^2\,dy\right]dy\right]$$

$$= \frac{b}{\pi}\left[\sin^{-1}\left(\frac{y}{a}\right)\left[\frac{y^3}{3}\right]_0^a - \int_0^a\left[\frac{1}{\sqrt{1-\left(\frac{y}{a}\right)^2}}\times\frac{1}{a}\left(\frac{y^3}{3}\right)\right]dy\right]$$

$$\boxed{\begin{aligned}\sqrt{a^2-y^2} &= t\\ a^2-y^2 &= t^2\\ -2y\,dy &= 2t\,dt\\ dy &= -\frac{t\,dt}{y}\end{aligned}}$$

$$= \frac{b}{\pi}\left[\frac{a^3}{3}\sin^{-1}\left(\frac{a}{a}\right) - \int_0^a\left[\frac{a}{\sqrt{a^2-y^2}}\times\frac{1}{a}\left(\frac{y^3}{3}\right)\right]dy\right]$$

$$= \frac{b}{\pi}\left[\frac{a^3}{3}\times\frac{\pi}{2} - \int_0^a\left[\frac{1}{\sqrt{a^2-y^2}}\cdot\frac{y^3}{3}\right]dy\right] = \frac{b}{\cancel{\pi}}\times\frac{\cancel{\pi}}{2}\times\frac{a^3}{3} - \frac{b}{3\pi}\int_0^a\left[\frac{1}{t}\times y^3\right]dy$$

$$= \frac{a^3b}{6} - \frac{b}{3\pi}\int_0^a\left(\frac{1}{\cancel{t}}\times \cancel{y^3}^{\,y^2}\times -\frac{\cancel{t}\,dt}{\cancel{y}}\right)$$

$$= \frac{a^3b}{6} + \frac{b}{3\pi}\int_0^a y^2\,dt = \frac{a^3b}{6} + \frac{b}{3\pi}\int_0^a (a^2-t^2)\,dt$$

$$= \frac{a^3b}{6} + \frac{b}{3\pi}\left[a^2t - \frac{t^3}{3}\right]_0^a = \frac{a^3b}{6} + \frac{b}{3\pi}\left[a^3 - \frac{a^3}{3}\right]$$

$$= \frac{a^3b}{6} + \frac{b}{3\pi}\left[\frac{2a^3}{3}\right] = \frac{a^3b}{6} + \frac{2a^3b}{9\pi} = 0.166a^3b + 0.0707a^3b$$

$$= 0.2367a^3b$$

* What are the conditions under which the centre of gravity of body becomes the same as its centroid?

A). Centroid of a body is same as its centre of gravity only if the ① acceleration due to gravity and ② The density of material is uniform throughout the body.

---

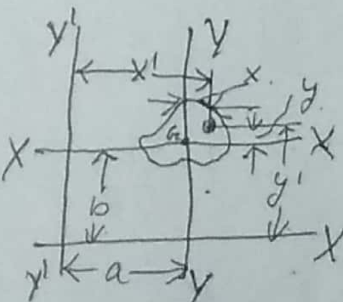
* State the transfer formula for product of Inertia?

A) When the product of inertia of plane area (or) figure is known about its centroidal axes XX and YY. It is possible to transform the moments of inertia about any other two axes X'X' and Y'Y' parallel to axes XX and YY.

Consider an area 'A' with axes. dA being small area (hatched)

$$I_{x'y'} = \int_A x'y'\,dA = \int_A (x+a)(y+b)\,dA$$

$$= \int_A (xy + xb + ay + ab)\,dA$$



$$= I_{xy} + 0 + 0 + abA$$

$$I_{x'y'} = I_{xy} + Aab.$$

This is the transfer formula for product of inertia

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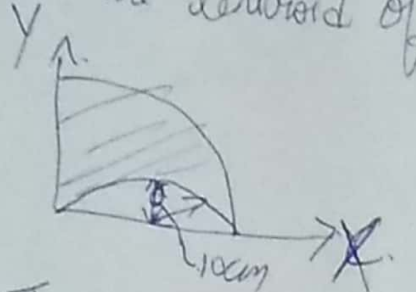
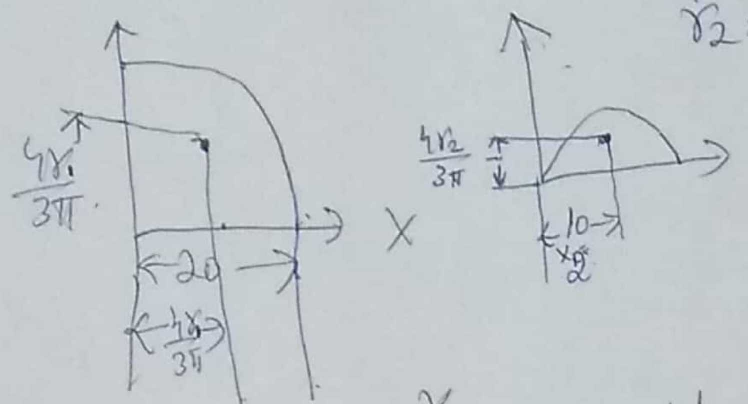
transfer Theorem means Parallel axis theorem

* Locate the centroid of hatched area ($r = 10cm$)



A) The given area can be considered as quarter circle area of radius 20, from a semicircular area of radius 10 cm is removed.

$r_1 = 20$
$r_2 = 10.$



$A_1 = \frac{\pi r_1^2}{4} = \frac{\pi \times 20^2}{4} = 314.15 mm^2$

$y_1 = x_1 = \frac{4r_1}{3\pi} = \frac{4 \times 20}{3\pi} = 8.48 mm$

$x_2 = 10$

$A_2 = \frac{\pi r_2^2}{2} = 157.07 mm^2$

$y_2 = \frac{4 \times 10}{3\pi} = 4.24 mm$

$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{314.15(8.48) - 157.07 \times 10}{314.15 - 157.07} = 6.96.$

$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = 12.71 mm$

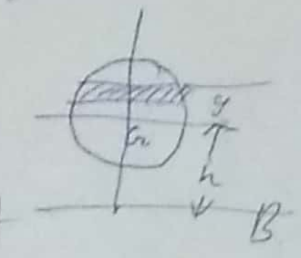
~~= 1.57~~

---

* State and prove parallel-axis theorem with legible sketch

A) It states, if the moment of inertia of a plane area about an axis through its Centre of gravity is denoted by I_G, then M.I of area about an axis AB, parallel to the first and at a distance h from C.g is given by

$$I_{AB} = I_G + ah^2$$



I_{AB} = M.I of area about an axis AB,
I_G = M.I of area about its C.g.
a = area of section
h = Distance b/w C.g of section and axis AB.

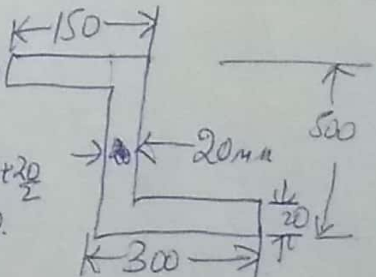
Proof:-
Consider a strip of circle, whose M.I is required to be found out about a line AB.
δa = Area of strip / y = Distance of strip from the C.g the section and h = distance b/w C.g of section and axis AB.
M.I of whole section about an axis passing through C.g of section $I_G = \Sigma \delta a y^2$.
∴ M.I of section about the axis AB,
$I_{AB} = \Sigma \delta a (h+y)^2 = \Sigma \delta a (h^2 + y^2 + 2hy) = I_G + ah^2$
$\Sigma h^2 \times \delta a = ah^2$ and $\Sigma y^2 \times \delta a = I_G$, $(\Sigma 2hy\,\delta a)$ is neglected.
$I_{AB} = ah^2 + I_G = \Sigma h^2 (\delta a) + (\Sigma y^2)\delta a$

* Differentiate b/w Centroid and centre of gravity?

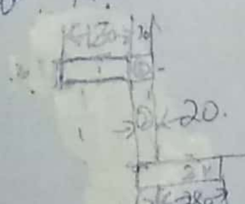
A)

Centroid (C)	Centre of gravity (C-g) or G
1) It is referred to the geometrical centre of a body.	1) The point where the total weight of body focusses upon.
2) The Centroid is a point in a plane area in such a way that moment of area about any axis throughout that point is 0	2) Centre of gravity in a uniformly gravitational field is the average of all the points, weighted by local density or Specific weight
3) It is ~~point~~ referred to Centre of gravity of uniform densed objects	3) It is a point where the gravitational force (weight) acts on the body.
4) It is a ~~physical~~ geometrical behaviour. It is centre of measure of amount of geometry	4) It is physical behaviour of object, a point where all the weight of object is acting

* Find the centroid of Z section



A) X_1



$y_1 = \frac{20}{2} = 10$ | $y_3 = 20 + 460 + \frac{20}{2} = 490.$

$y_2 = 20 + \frac{460}{2} = 250$

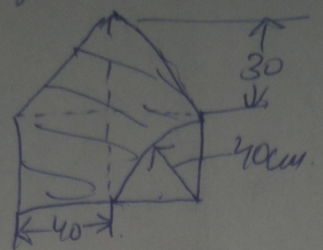
$X_1 = 130 + \frac{300}{2} = 280$, $X_2 = 150 - \frac{20}{2} = 140$, $X_3 = \frac{150}{2} = 75$, $A_1 = 20 \times 300 = 6000 mm^2$

$A_2 = 460 \times 20 = 9200 mm^2$, $A_3 = 150 \times 20 = 3000 mm^2$

$\bar{x} = 175.4 mm$

$\bar{y} = 210.4 mm$

* Find the M.I of shaded area about centroidal axis



A). Area of Rectangle $\boxed{}$ 40, 80. | $x_1 = \frac{80}{2} = 40$ | $y_1 = \frac{40}{2} = 20$

$= 80 \times 40 = 3200\,mm^2$

Area of triangle $= \frac{1}{2} \times 80 \times 30$ (△ 80, 30) | $x_2 = \frac{40+80}{3} = 40mm$

$y_2 = 40 + \frac{30}{3} = 50mm$

$= 1200\,mm^2$

Area of quarter circle (D cle) $= \frac{1}{4} \times \pi r^2 = \frac{1}{4} \times \pi \times 40^2 = 1256.63$

$x_3 = 80 - \frac{4(40)}{3\pi} = 63.02$

$\bar{y}_3 = \frac{4R}{3\pi} = \frac{4 \times 40}{3\pi} = 16.97$

$\bar{X} = 30.7,\ \bar{y} = 32.66$

$(I_{xx}$ of rectangle$) = \frac{bd^3}{12} = \frac{80 \times 40^3}{12} = 426666.66\,cm^4$

$(I_{yy}$ of rectangle$) = \frac{db^3}{12} = 1706666.66\,cm^4$

$(I_{xx}$ of △le$) = \frac{bd^3}{36} = \frac{80 \times 30^3}{36} = 60000$ | $I_{yy} = \frac{db^3}{36} = 426666.66\,cm^4$

$(I_{xx}$ of D circle$) = I_{yy} = 0.0549r^4 = 140544\,mm^4$

$\bar{x}_1 = \underset{(\bar{x} - x_1)}{40 - 30.7} = 9.3$, $\bar{x}_2 = \underset{(\bar{x} - x_2)}{40 - 30.7} = 9.3$, $\bar{x}_3 = 63.02 - 30.7 = \underset{(\bar{x} - x_3)}{32.32}$

$\bar{y}_1 = \bar{y} - y_1 = 32.66 - 20 = 12.66mm$ | $\bar{y}_2 = y_2 - \bar{y} = 50 - 32.66 = 17.34$

$\bar{y}_3 = \bar{y} - y_3 = 32.66 - 16.97 = 15.69$

$I_{\text{centroidal axis }(x-x)} = ((I_{xx}$ of □le + Area of □le $(\bar{x}_1)^2) +$
$(I_{xx}$ of △le + Area of △le $(\bar{x}_2)^2) -$
$(I_{xx}$ of Dcle + Area of Dcle $(\bar{x}_3)^2)$

$= -585974.92\,cm^4$

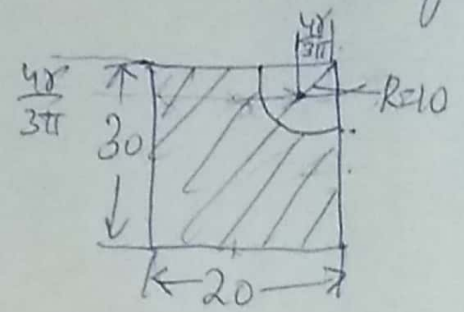
Similarly I_{yy} axis

$= (I_{xx}$ of □le + Area of □le $(\bar{y}_1)^2) + (I_{yy}$ of △le + Area of △le $(\bar{y}_2)^2)$
$- (I_{yy}$ of Dcle + Area of Dcle $(\bar{y}_3)^2)$

$= 2557,129.687\,cm^4 = 2.5 \times 10^6\,cm^4$

* Find the Centroid of area. All dimensions are in cm

A)



A) Area of ▭ $= 30 \times 20 = 600\,cm^2$

$x_1 = \frac{20}{2} = 10$

$y_1 = \frac{30}{2} = 15$

Area of Quarter circle $= \frac{\pi r^2}{4} = \frac{1}{4} \times \pi \times 10^2 = 78.539$

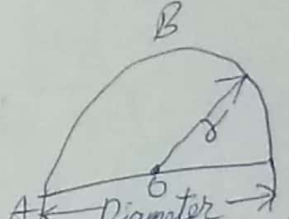
$x_2 = 20 - \frac{4 \times 10}{3\pi} = 15.75$, $y_2 = 30 - \frac{4 \times 10}{3\pi} = 25.75$

$\bar{x} = 9.13$, $\bar{y} = 13.38$

---

* Derive an expression to determine the M.I of a semicircle about its diametric base

A) Consider a semicircle ABC.



r = Radius of semicircle.

M.I of semicircle about base AC is equal to the half of M.I of circular section about AC.

Therefore M.I of semicircle section about base AC

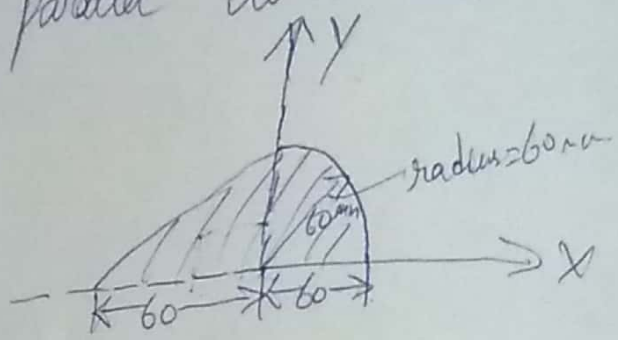
$I_{AC} = \frac{1}{2} \times \frac{\pi}{64} \times (d)^4 = 0.393 r^4$. $(d = 2r)$

Area of semicircle $= \frac{\pi r^2}{2} = \frac{\pi \left(\frac{d}{2}\right)^2}{2} = \frac{\pi d^2}{8}$

Distance b/w Centre of gravity and base AC

$h = \frac{4r}{3\pi}$

* Find M.I of shaded area. about its Centroidal axes parallel to X-axes



A)

Segment	Area	x	y	Ax	Ay
Quarter circle	$\frac{\pi R^2}{4}$ $= 2827.43$	$\frac{4R}{3\pi}$ $x_1 = 25.46$	$\frac{4R}{3\pi}$ $y_1 = 25.46$	72000	72000.
Triangle	$\frac{bh}{2}$ $= 1800$	$\frac{-h}{3}$ $x_2 = -20$	$+\frac{b}{3}$ $y_2 = +20$	-36000	36000

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{72000 - 36000}{2827.43 + 1800} = 7.8.$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{72000 + 36000}{2827.43 + 1800} = 23.33$$

$$I_{xx} = \left(\underbrace{0.0549r^4}_{\text{M.I of quater circle}} + \underbrace{A}_{\text{Area of Quarter circle}}(\bar{y}_1)^2\right) + \left(\underbrace{\frac{bh^3}{36}}_{\text{M.I of }\Delta le} + \underbrace{A}_{\text{Area of }\Delta le}(\bar{y}_2)^2\right)$$

$\bar{x}_1 = x_1 - \bar{x} = 25.46 - 7.8$
$= 17.66.$

$\bar{x}_2 = x_2 - \bar{x} = -20 - 7.8$
$= -27.8$

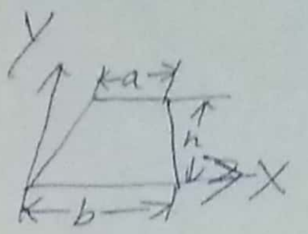
$\bar{y}_1 = y_1 - \bar{y} = 25.46 - 23.33$
$= 2.13$

$\bar{y}_2 = \bar{y} - y_2 = 23.33 - 20$
$= 3.33$

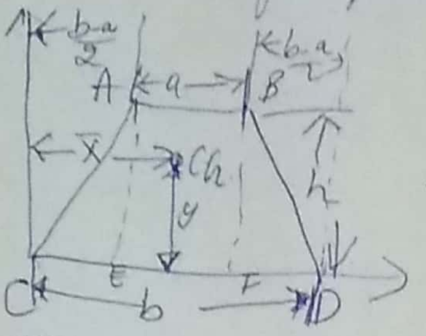
$$I_{\bar{x}\bar{x}} = \left(0.0549(60)^4 + 2827.43(2.13)^2\right) + \left(\frac{60 \times 60^3}{36} + 1800(3.33)^2\right)$$

$= \cancel{1102611.787}$ $1104291.803\ mm^4$

* Derive the Centroid of trapezium

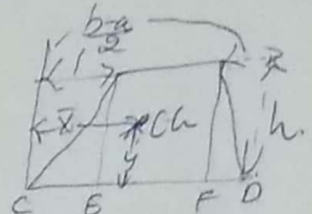


A).



trapezium is made up of 2Δles & rectangle.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$



A_1 = Area of Δle ACE $= \frac{1}{2} \times CE \times AE = \frac{1}{2} \times \frac{b-a}{2} \times h$

A_3 = Area of Δle BFD $= \frac{1}{2} \times BF \times FD = \frac{1}{2} \times \frac{b-a}{2} \times h$

$y_1 = \frac{1}{3}h, \; y_2 = \frac{1}{2}h, \; y_3 = \frac{1}{3}h.$

Area of ▭le BAFE $= a \times h$

$$\bar{y} = \frac{\frac{1}{2} \times \frac{b-a}{2} \times h \times \frac{h}{3} + a \times h \times \frac{h}{2} + \frac{1}{2} \times \frac{b-a}{2} \times h \times \frac{h}{3}}{\frac{1}{2} \times \frac{b-a}{2} \times h + ah + \frac{1}{2} \times \frac{b-a}{2}(h)}$$

$$= \frac{\frac{b-a}{2} \times \frac{h^2}{3} + \frac{ah^2}{2}}{\frac{b-a}{2}h + ah} = \frac{h}{3}\left[\frac{b+a+a}{b+a}\right]$$

$$\bar{x} = \frac{b}{2}$$

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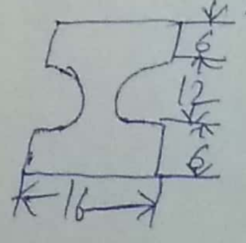
* Find Moments of Inertia of cut section shown about Centroidal axis. Two semicircular portions are cut from rectangular plate.



A) As the section is symmetrical about its horizontal and vertical axis, therefore C.g of section will lie at centre of rectangle

radius of circle $= \frac{12}{2} = 6$

M.I of the section about horizontal axis passing through centroid of the section $(b=16, d=24)$

$$= \frac{bd^3}{12} = \frac{16 \times 24^3}{12} = 18432\ mm^4$$

M.I of circular section about a horizontal axis passing through its C.g. $= \frac{\pi}{4} r^4 = \frac{\pi}{4} \times 6^4 = 1017.87\ mm^4$

M.I of whole section about horizontal axis passing through centroid of section

$$I_{xx} = 18432 - 1017.87 = 17414.13\ mm^4$$

M.I of section about vertical axis passing through centroid of section

$$I_{G_1} = \frac{db^3}{12} = \frac{24 \times 16^3}{12} = 8192$$

area of semicircular section with 6mm radius

$$A = \frac{\pi r^2}{2} = \frac{\pi \times 6^2}{2} = 56.54\ mm^2$$

M.I of semicircular section about vertical axis passing through C.g. $I_{G_2} = 0.11 r^4 = 0.11 \times 6^4 = \cancel{\ldots}$ $142.56\ mm^4$

Distance b/w C.g of semicircular section and its base.

$$= \frac{4r}{3\pi} = \frac{4 \times 6}{3\pi} = 2.54\ mm$$

∴ Distance b/w C.g of semicircular and C.g of whole section

$h_2 = \cancel{ao}\ \frac{16}{2} - 2.54 = 5.46$ mm

M.I of one semicircular section about cg of whole section $= I_{G_2} + \underbrace{a_2}_{\text{area of semi circle}} h_2^2 = 142.56 + (56.54)(5.46)^2$

$= 1828.10\ mm^4$

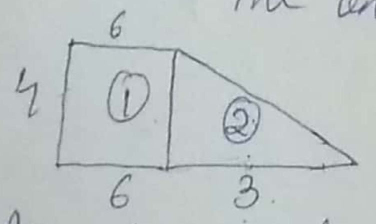
M.I of two semicircular section about C.g of whole section $= 2(1828.10) = 3656.2$.

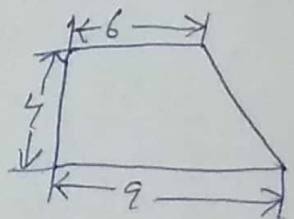
M.I of whole section about vertical axis (I_{YY}) passing through centroid of section

$= 8192 - 3656.2 = 4535.8\ mm^4$

* Determine the centroid of area

A)





Area of rectangle $= 6 \times 4 = 2\ cm^2$

$x_1 = \frac{6}{2} = 3\ cm$ $y_1 = \frac{4}{2} = 2\ cm$

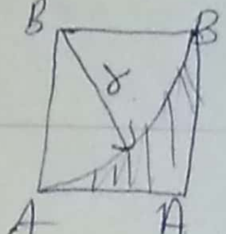
Area of $\Delta le = \frac{1}{2} \times 4 \times 3 = 6\ cm^2$

$x_2 = 6 + \frac{3}{3} = 7\ cm$ $y_2 = \frac{4}{3} = 1.33\ cm$

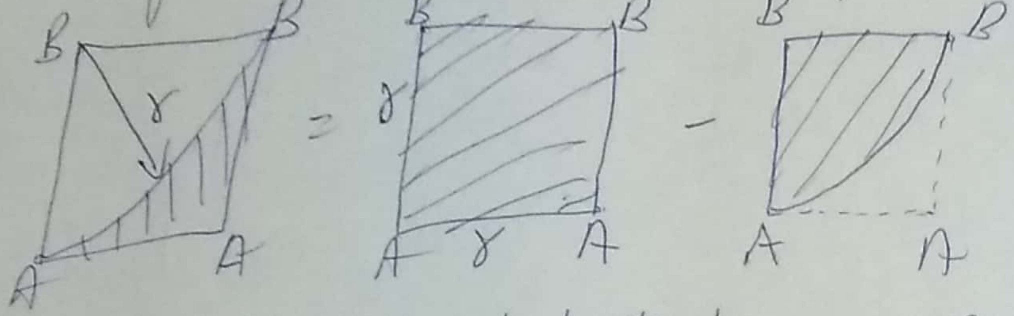
$\bar{X} = \frac{2 \times 3 + 6 \times 7}{2+6} = \cancel{6cm}\ 6\ cm$

$\bar{Y} = \frac{2 \times 2 + 6 \times 1.33}{2+6} = 1.5\ cm$

* Determine the M.I of quarter circle spandrel about axes AA and about BB



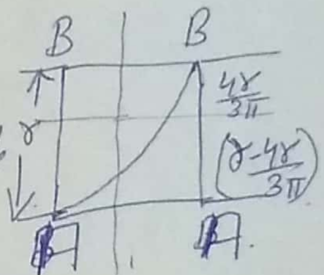
A). The shaded area can be obtained by removing a quarter circular area of radius r from a square of side r



M.I of square about its base is

$$I_a + A(h)^2 = I_{\text{Base of Square}}$$

$$\frac{r^4}{12} + r^2\left(\frac{r}{2}\right)^2 = 0.33r^4$$



h - Distance b/w centroid to its base $= \frac{r}{2}$

M.I of quarter circular area about its centroidal axes

$$I_2 = 0.0549(r)^4$$

M.I of quarter circular about AA axes is

$$I_2 = 0.0549(r)^4 + \frac{\pi}{4}r^2\left(r - \frac{4r}{3\pi}\right)^2 = 0.315r^4.$$

M.I of spandrel about AA is obtained as

$$I_{AA} = I_1 - I_2 = 0.33r^4 - 0.315r^4 = 0.015r^4$$

Similarly M.I of quarter circular area about BB axes.

$$I_2' = 0.0549(r)^4 + \frac{\pi}{4}r^2\left(\frac{4r}{3\pi}\right)^2 = 0.196r^4.$$

M.I of spandrel about BB is $I_{BB} = I_1 - I_2' = 0.33r^4 - 0.198r^4 = 0.134r^4$

* ~~Determine~~ Explain why First moment of an area with an axis of Symmetry is zero.

A) The First moment of an area about any Centroid axis of the area are zero. ~~Since~~ as the Centroid is located on centroidal axis, the perpendicular distance from centroid to the centroidal axis must be zero.

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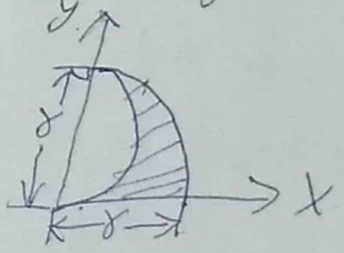
* Determine the centroid of shaded area formed by removing a semicircle of diameter (r) from a Quadric circle of radius r.

A). <u>For the quarter circle</u>

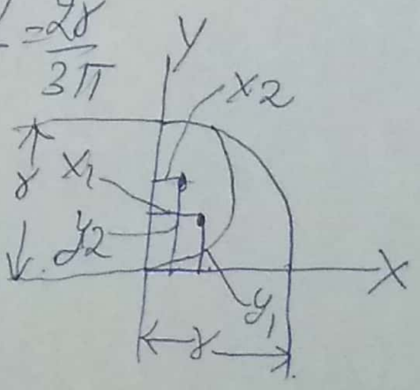


$A_1 = \frac{\pi r^2}{4}$ | $x_1 = \frac{4r}{3\pi}$ | $y_1 = \frac{4r}{3\pi}$

<u>For the semi circle</u>

$A_2 = \frac{1}{2}\left(\frac{\pi r^2}{4}\right) = \frac{1}{8}\pi r^2$ | $x_2 = \frac{4\left(\frac{1}{2}r\right)}{3\pi} = \frac{2r}{3\pi}$

$y_2 = \frac{1}{2}(r)$



<u>For shaded area</u>

$A = A_1 - A_2 = \frac{\pi r^2}{4} - \frac{\pi r^2}{8} = \frac{\pi r^2}{8}$

$\bar{x} = \Sigma a x / A$

$\bar{x} = \frac{\left(\frac{1}{4}\pi r^2\right)\left(\frac{4r}{3\pi}\right) - \left(\frac{1}{8}\pi r^2\right)\left(\frac{2r}{3\pi}\right)}{\frac{\pi r^2}{8}} = 0.6333r$

$$\bar{y} = \Sigma a y / A = \frac{\frac{\pi r^2}{4}\left(\frac{4r}{3\pi}\right) - \frac{\pi r^2}{8}\left(\frac{-r}{2}\right)}{\frac{\pi r^2}{8}} = 0.34888r$$

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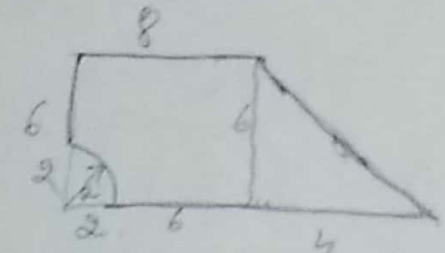
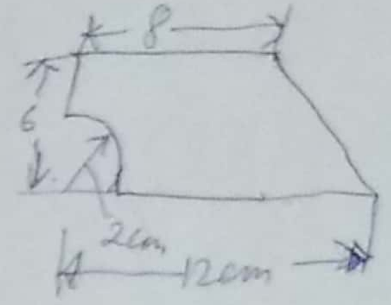
* Compute the M.I (Second moment of area)

A)





Area of rectangle = 8×6=48.

$x_1 = \frac{8}{2} = 4$, $y_1 = \frac{6}{2} = 3$ cm

Area of Δle = $\frac{1}{2} \times 6 \times 4 = 12$

$x_2 = 8 + \frac{4}{3} = 9.333 = \frac{28}{3}$ | $y_2 = \frac{6}{3} = 2$

Area of quarter circle = $\frac{1}{4} \times \pi (2)^2 = \pi$ cm

$r = 2$.

$x_3 = \frac{4\times2}{3\pi} = \frac{8}{3\pi}$ | $y_3 = \frac{4\times2}{3\pi} = \frac{8}{3\pi}$

$$\bar{X} = \frac{48\times4 + 12\times\frac{28}{3} - \pi\times\frac{8}{3\pi}}{48+12-\pi} = 53 \quad \Big| \quad \bar{y} = 2.9$$

$$I_{xx} = \left(\frac{bd^3}{12} + A(\bar{x}-x_1)^2\right)_{\text{rectangle}} + \left(\frac{bd^3}{36} + A(x_2-\bar{x})^2\right)_{\Delta le} - \left(0.0549r^4 + A(\bar{x}-x_3)^2\right)$$

$r=2$, $A=\pi$

$b=8, d=6, A=48$ $b=4, d=6, A=12$

$I_{xx} = 38120 \text{ mm}^4$

Similarly

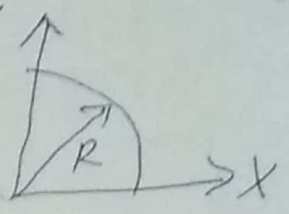
$$I_{yy} = \left(\frac{db^3}{12} + A(y_1-\bar{y})^2\right) + \left(\frac{db^3}{36} + A(\bar{y}-y_2)^2\right) - \left(0.0549r^4 + A(\bar{y}-y_3)^2\right)$$

$d=6, b=8, A=48$ $d=6, b=4$ $A=12$ π

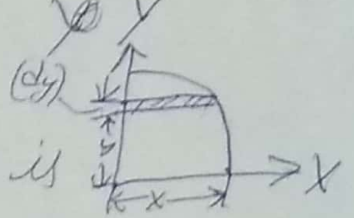
$= 262.76 \text{ mm}^4$

* Determine the product of Inertia of the quarter circular area with respect to given X and Y axes

A) Consider a thin strip parallel to X-axis at a distance (y) from the X-axis and of infinitesimally small thickness (dy). Then area of strip is



$$dA = x\,dy$$



Product of Inertia of this strip is

$$dI_{xy} = \left(\frac{x}{2}\right) y\,dA = \frac{x}{2} y x\,dy = \frac{x^2 y}{2} dy$$

Here strip can be considered to be rectangle and its centroid lies at a midpoint.

$\therefore$ Product of Inertia of entire area X-Y axes is

$$I_{xy} = \int_0^R \left(\frac{x}{2}\right) yx\,dy = \int_0^R \frac{x^2}{2} y\,dy$$

$x^2+y^2=R^2$ (Equation of circle) $x^2=R^2-y^2$

$$I_{xy} = \frac{1}{2}\int_0^R (R^2-y^2)\,y\,dy = \frac{1}{2}\int_0^R (R^2y-y^3)\,dy$$

$$= \frac{1}{2}\left(\frac{R^4}{2}-\frac{R^4}{4}\right) = \frac{R^4}{8}$$